Characterization and Greedy Learning of Interventional Markov Equivalence Classes of Directed Acyclic Graphs (Abstract)

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Introduction

Directed acyclic graphs (DAGs) are commonly used to model causal relationships between random variables. Markov equivalence of DAGs indicates to which extent these causal influences are identifiable from the *observational* density of the random variables. In this paper, we extend the notion of Markov equivalence of DAGs to the case of *interventional* distributions arising from *multiple* intervention experiments which are crucial for improved causal inference, and we generalize the greedy equivalence search (GES) algorithm in order to process observational and interventional data (or data from different interventions) *simultaneously*.

Model

We consider causal models of the form (D, f), where D is a DAG with p vertices and f a p-variate probability density of random variables $X_1, \ldots, X_p$ obeying the *Markov property* of D. An *intervention* is an experiment that forces the value of one or several of the random variables (called *intervention target*) to chosen values, destroying their original dependencies. The effect of an intervention is formalized by the do-calculus (Pearl, 2009). We consider data arising from *multiple* intervention experiment performed at different intervention targets, which lead to better identifiability compared to the observational case.

Main Theoretical Results

We present a graph theoretic criterion for two DAGs being Markov equivalent under a given set of different intervention targets which reproduces the classical result of Verma and Pearl (1992) in the special case of purely observational data.

Furthermore, we show that each interventional Markov equivalence class can, analogously to the observational case, be uniquely represented by a partially directed graph called *essential graph* (also known as *CPDAG* in the observational case). We give a graph theoretic characterization of interventional essential graphs which reduces to the one of Andersson et al. (1997) in the observational case.

Structure Learning

Based on our characterization of interventional essential graphs, we derive a generalization of the GES algorithm of Chickering (2002) aimed at structure learning from *interventional* data, yielding an algorithm called greedy interventional equivalence search (GIES) which can be used for regularized maximum likelihood estimation.

Simulations

We evaluate GIES on interventional data simulated from 5000 randomly generated Gaussian causal models. GIES markedly beats the conceptually simpler greedy search over the space of DAGs and exploits the substantially improved identifiability thanks to interventions. GIES keeps up with a provably consistent exponential time dynamic programming algorithm at much lower computational costs.

References

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Full Text

The full paper is available at
`http://arxiv.org/abs/1104.2808`